MAT 261—Exam #3—4/17/14

Name: ____

Calculators are not permitted. Show all of your work using correct mathematical notation.

1. (15 points) Consider the integral $\int_0^6 \int_0^4 (x + 2y)\, dy\, dx$.

   (a) Find the Riemann sum approximation $S_{2,2}$ to the integral, using 4 rectangles with $\Delta x = 3$ and $\Delta y = 2$ and the lower left vertices as sample points.

   (b) Find the exact value of the integral.

2. (15 points) Find the average value of the function $f(x, y, z) = \dfrac{e^{2z}}{(x+3y)^2}$ over the box $[1, 5] \times [0, 2] \times [0, 1]$.

3. (15 points) Evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(\pi y^3)\, dy\, dx$ by reversing the order of integration. Include a sketch of the domain.

4. (15 points) Evaluate the integral $\int_0^2 \int_2^{\sqrt{8-x^2}} (x^2+y^2)^{-3/2}\, dy\, dx$ by changing to polar coordinates. Include a sketch of the domain.

5. (15 points) An object occupying the hemisphere defined by $x^2+y^2+z^2 \leqslant 4$ and $z \geqslant 0$ has mass density $\delta(x,y,z) = 3z^2$ kg per cubic unit. Find the total mass of the object.

6. (15 points) Consider the integral $\iint_{\mathcal{D}} (y-x)^5 \, dA$, where $\mathcal{D}$ is the parallelogram in the xy-plane spanned by the vectors $\langle 4, 5\rangle$ and $\langle 1, 3\rangle$. Use the transformation

$$\Phi(u, v) = (4u + v, 5u + 3v)$$

to evaluate the integral.

7. (10 points) Evaluate the line integral $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s}$ for the vector field $\mathbf{F} = \langle x^2, xy\rangle$ and the curve $\mathcal{C}$ parametrized by $\mathbf{c}(t) = \langle t^3, 2t\rangle$ on the interval $0 \leqslant t \leqslant 1$.