MAT 261—Exam #3—11/13/14

Name: ____

Calculators are not permitted. Show all of your work using correct mathematical notation.

1. (15 points) Find the average value of the function $f(x, y) = x + \sqrt{y}$ over the triangle bounded by the lines $y = 0$, $x = 1$, and $y = x$.

2. (10 points) Consider the integral $\displaystyle\int_0^2 \int_1^{e^x} f(x, y)\, dy\, dx$. Sketch the domain of integration, and set up an equivalent integral with the order of integration reversed.

3. (10 points) Set up (**but do not evaluate**) an integral that gives the volume of a solid whose base is the region in the xy-plane between the curves $y = x^2$ and $x = y^2$ and whose upper boundary is the elliptical paraboloid $z = 9 - x^2 - 2y^2$.

4. (15 points) Evaluate the integral $\int_0^1 \int_{\sqrt{3}\,x}^{\sqrt{4-x^2}} (x^4 y + x^2 y^3)\, dy\, dx$ by changing to polar coordinates. Include a sketch of the domain.

5. (15 points) Evaluate the triple integral $\displaystyle\int_0^1 \int_0^2 \int_0^1 \frac{yz^4 \sin(\pi x)}{3+y^2}\, dz\, dy\, dx.$

6. (15 points) Consider the integral $\displaystyle\iint_{\mathcal{D}} (x+y)\, dA$, where $\mathcal{D}$ is the parallelogram in the xy-plane spanned by the vectors $\langle 5, 2\rangle$ and $\langle 1, 3\rangle$. Use the transformation

$$G(u,v) = (5u+v, 2u+3v)$$

to evaluate the integral.

7. (20 points) An object occupying the region defined by the inequalities $x^2 + y^2 + z^2 \leqslant 18$ and $z \geqslant 3$ has mass density $\delta(x, y, z) = 5/z$ kg per cubic unit. Set up (**but do not evaluate**) integrals that give the mass of the object:

   (a) using cylindrical coordinates

   (b) using spherical coordinates