MAT 162—Practice Final Exam A—Spring 2011

1. Evaluate the integral $\displaystyle\int x^5 \ln x \, dx$.

2. Evaluate the integral $\displaystyle\int \frac{dx}{x\sqrt{16-x^2}}$.

3. Evaluate the integral $\displaystyle\int \frac{dx}{x^3+4x}$.

4. Find the area between the curves $y = e^x$ and $y = e^{2x}$ on the interval $[0, 1]$.

5. Find the volume of the solid obtained by revolving the region bounded by the curve $y = x^3 + 1$ and the lines $x = 0$, $x = 1$, and $y = 0$ about the y-axis.

6. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \pi^{-n/2}$, or show that it diverges.

7. Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 2^n}$.

8. Let $a_n = \dfrac{n^2+4}{3n^2}$. Evaluate $\displaystyle\lim_{n\to\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} a_n$.

9. Find the length of the curve $y = 2x^{3/2}$ between $x = 0$ and $x = 1$.

10. A tank has the shape of a cone, with vertex at the bottom, of height 12 meters and radius 5 meters. The tank is filled to a height of 10 meters with water. Set up (but do not evaluate) an integral that gives the work required to pump all the water out through a spout that extends 1 meter above the tank's top.

11. Evaluate the improper integral $\displaystyle\int_0^{\infty} \frac{x^2}{(x^3+1)^4}\, dx$, or show that it diverges.

12. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+5^n}$ converges absolutely, converges conditionally, or diverges, and justify your answer.

13. Find the Taylor polynomial of order 3 centered at $x = 1$ for the function $f(x) = \sqrt{x}$, and compute the approximation to $\sqrt{1.1}$ given by this polynomial.

14. Use the first three non-zero terms of a series to estimate $\displaystyle\int_0^1 \cos(x^3)\, dx$, and give an upper bound for the error in your estimate.

15. For what values of p does the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)(\ln \ln n)^p}$ converge? Justify your answer.

MAT 162—Practice Final Exam B—Spring 2011

1. Evaluate the integral $\int xe^{5x}\,dx$.

2. Evaluate the integral $\int \frac{x^3\,dx}{\sqrt{9+x^2}}$.

3. Evaluate the integral $\int \frac{dx}{x^2+4x+3}$.

4. Find the volume of the solid obtained by revolving the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$ about the x-axis.

5. Find the average value of the function $f(x) = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$.

6. Find the limit of the sequence $a_n = (1 + 1/n)^n$, or show that it diverges.

7. Determine the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{5^n}$.

8. True or False? If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

9. Use the Midpoint Rule with $N = 5$ subdivisions to approximate $\int_0^2 x^3\,dx$, and give an upper bound for the error in your estimate.

10. Find the area under the curve $y = \cos^3 x$ between $x = 0$ and $x = \pi/2$.

11. Set up (but do not evaluate) an integral that gives the volume of the solid obtained by revolving the region bounded by the curve $y = e^{-x}$ and the lines $y = 1$ and $x = 1$ about the line $x = 2$.

12. A semi-circular plate, with diameter along the top, is submerged vertically in water. The plate's radius is 6 feet, and the top of the plate is 2 feet below the surface of the water. Set up (but do not evaluate) an integral that gives the hydrostatic force on one side of the plate.

13. Determine whether the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$ converges absolutely, converges conditionally, or diverges, and justify your answer.

14. Use the first three non-zero terms of a series to estimate $\int_0^{1/3} e^{-x^2}\,dx$, and give an upper bound on the error in your estimate.

15. Show that the object obtained by revolving the portion of the curve $y = 1/x$ with $x \ge 1$ about the x-axis has infinite surface area.

MAT 162—Formula Sheet for Final Exam—Spring 2011

Work = Force $\times$ Distance

Pressure = Weight Density $\times$ Depth

Hydrostatic Force = Pressure $\times$ Area

Weight density of water: 62.4 lb/ft^3 or 9800 N/m^3

$$\sin 2u = 2\sin u\cos u \qquad \sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2}$$

$$\int \tan ax\,dx = \frac{1}{a}\ln|\sec ax| + C \qquad \int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \cot ax\,dx = \frac{1}{a}\ln|\sin ax| + C \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \sec ax\,dx = \frac{1}{a}\ln|\sec ax + \tan ax| + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \csc ax\,dx = -\frac{1}{a}\ln|\csc ax + \cot ax| + C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$$

If $|f''(x)| \le K_2$ on $[a,b]$ then $|E_T| \le \dfrac{K_2(b-a)^3}{12N^2}$ and $|E_M| \le \dfrac{K_2(b-a)^3}{24N^2}$.

$$S_N = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)\right)$$

If $|f^{(4)}(x)| \le K_4$ on $[a,b]$ then $|E_S| \le \dfrac{K_4(b-a)^5}{180N^4}$.

Taylor Series for $f(x)$ centered at $x=c$: $\quad f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x|<1 \qquad e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \quad \text{for all } x$$

$$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad \text{for all } x \qquad \cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!} \quad \text{for all } x$$

$$(1+x)^a = 1 + \sum_{n=1}^{\infty}\binom{a}{n}x^n \quad \text{for } |x|<1, \quad \text{where} \quad \binom{a}{n} = \frac{a(a-1)(a-2)\cdots(a-n+1)}{n!}$$

If $|f^{(n+1)}(u)| \le K$ for u between c and x then $|f(x) - T_n(x)| \le \dfrac{K}{(n+1)!}|x-c|^{n+1}$.