MAT 162—Exam #1—2/20/15

Name: _____

Show all work using correct mathematical notation. Calculators are not allowed.

1. (15 points) Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[1, 4]$.

2. (10 points) Set up (**but do not evaluate**) a definite integral that gives the length of the curve $y = \sin(2x)$ from $x = 0$ to $x = \pi$.

3. (15 points) Find the volume of the solid obtained by revolving the region bounded by the curve $y = e^{3x}$ and the lines $x = 0$, $y = 0$, and $x = 1$ about the x-axis.

4. (15 points) Set up (**but do not evaluate**) definite integrals that give the volumes of the solids obtained by revolving the region bounded by the curves $y = 1/x$, $y = x$, and $x = 2$ about the given axes. In each case, show a representative disk, washer, or shell on the sketch provided.

   (a) the y-axis
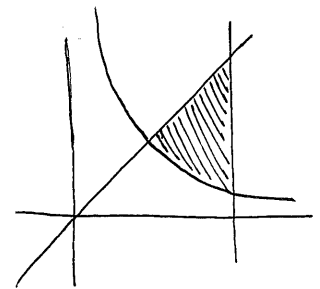
   (b) the line $y = 3$
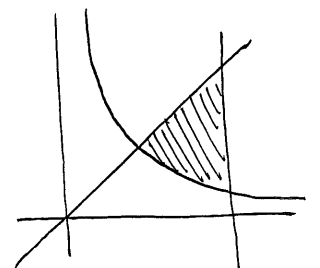
5. (15 points) Sketch the region bounded by the line $y = x - 6$ and the parabola $x = y^2$, and label the points of intersection. Then express the area of the region using one or more definite integrals. **Do not evaluate the integral(s).**

6. (15 points) A conical tank of radius 7 meters and height 10 meters is filled to a height of 9 meters with water, which weighs 9800 N/m^3. Water is to be pumped out through a spout that extends 2 meters above the tank's top.

(a) Find the weight of a slice of thickness Δy located at y meters from the bottom of the tank. Your answer should be expressed in terms of the variable y, as labeled in the diagram.
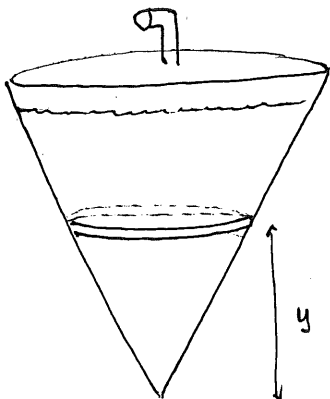

(b) Find the distance moved by the slice discussed in part (a) to reach the top of the spout.

(c) Set up (**but do not evaluate**) a definite integral that gives the total work required to empty the tank.

7. (15 points) A semi-circular plate of radius 8 feet is submerged in water, which weighs 62.4 lb/ft^3. The diameter of the plate lies 3 feet below the surface.

(a) Find the area of the strip of thickness Δy located at y feet below the top of the plate. Your answer should be expressed in terms of the variable y, as labeled in the diagram.

3

y
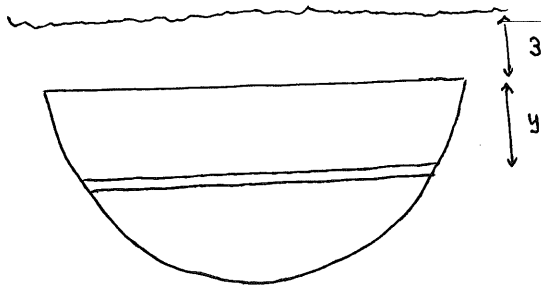
(b) Find the pressure along the strip discussed in part (a).

(c) Set up (**but do not evaluate**) a definite integral that gives the hydrostatic force on the plate.