MAT 161-03—Exam #2—10/19/10

Name: Solutions

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (20 points) For each function below, calculate the derivative. There is minimal simplification to do, but give your final answers in clean form.

   (a) $f(x) = 8 + \sqrt[3]{x}$

   $$f'(x) = \frac{1}{3} x^{-2/3}$$

   (b) $g(t) = 7e^{5t}$

   $$g'(t) = 7e^{5t} \cdot 5$$
   $$= 35e^{5t}$$

   (c) $h(x) = (\ln x)^4$

   $$h'(x) = 4(\ln x)^3 \cdot \frac{1}{x}$$
   $$= \frac{4(\ln x)^3}{x}$$

   (d) $r(\theta) = \theta^3 \sin\theta$

   $$r'(\theta) = \theta^3 \cos\theta + (\sin\theta) \cdot 3\theta^2$$
   $$= \theta^2(\theta\cos\theta + 3\sin\theta)$$

2. (20 points) Calculate $f'(x)$ for each of the following functions, and simplify your answers as much as possible.

(a) $f(x) = \dfrac{2x-5}{x+1}$

$$f'(x) = \frac{(x+1)\cdot 2 - (2x-5)\cdot 1}{(x+1)^2}$$

$$= \frac{7}{(x+1)^2}$$

(b) $f(x) = \ln(\sec x)$

$$f'(x) = \frac{1}{\sec x} \cdot \sec x \tan x$$

$$= \tan x$$

(c) $f(x) = xe^{x^2}$

$$f'(x) = x e^{x^2} \cdot 2x + e^{x^2} \cdot 1$$

$$= e^{x^2}(2x^2 + 1)$$

(d) $f(x) = \tan^{-1}(x^3)$

$$f'(x) = \frac{1}{(x^3)^2 + 1} \cdot 3x^2$$

$$= \frac{3x^2}{x^6 + 1}$$

3. (20 points) Calculate $\frac{dy}{dx}$ for each of the following functions. You do NOT need to simplify your answers, but be sure to include any required parentheses!

(a) $y = \cot(2^x \log_7 x)$

$$\frac{dy}{dx} = \left(-\csc^2\left(2^x \log_7 x\right)\right) \cdot \left(2^x \cdot \frac{1}{x \ln 7} + (\log_7 x) \cdot 2^x \ln 2\right)$$

(b) $y = \dfrac{(\sin^{-1} x)^{10}}{x^4 - x + 1}$

$$\frac{dy}{dx} = \frac{(x^4 - x + 1) \cdot 10\left(\sin^{-1} x\right)^9 \cdot \frac{1}{\sqrt{1-x^2}} - \left(\sin^{-1} x\right)^{10}\left(4x^3 - 1\right)}{\left(x^4 - x + 1\right)^2}$$

(c) $y = \cos^5(\tan x)$

$$\frac{dy}{dx} = 5\left[\cos(\tan x)\right]^4 \left(-\sin(\tan x)\right) \cdot \sec^2 x$$

(d) $y = \sqrt{\ln(\ln(\ln x))}$

$$\frac{dy}{dx} = \frac{1}{2}\left(\ln(\ln(\ln x))\right)^{-1/2} \cdot \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

4. (15 points) Find the slope of the tangent line to the curve $x^5 + 3x^2y + y^4 = 23$ at the point $(1, 2)$.

$$5x^4 + 3\left(x^2 \frac{dy}{dx} + y \cdot 2x\right) + 4y^3 \frac{dy}{dx} = 0$$

$$\Rightarrow \quad 5x^4 + 3x^2 \frac{dy}{dx} + 6xy + 4y^3 \frac{dy}{dx} = 0$$

$$\Rightarrow \quad (3x^2 + 4y^3) \frac{dy}{dx} = -5x^4 - 6xy$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-5x^4 - 6xy}{3x^2 + 4y^3}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-5-12}{3+32}$$

$$= \frac{-17}{35}$$

5. (5 points) Compute $f''(\pi/3)$ for the function $f(x) = \cos(3x)$.

$$f'(x) = -3\sin(3x)$$

$$f''(x) = -9\cos(3x)$$

$$f''(\pi/3) = -9\cos(\pi)$$

$$= -9(-1)$$

$$= 9$$

6. (10 points) The vertical position of a rock thrown into the air from a the top of a building is given by $s(t) = 40 + 32t - 16t^2$, where s is measured feet and t is measured in seconds. Find the maximum height reached by the rock.

Max height occurs when

$$v(t) = s'(t) = 32 - 32t = 0$$

$$\Leftrightarrow \quad t = 1$$

Hence the max height is

$$s(1) = 40 + 32 - 16$$

$$= 56 \text{ feet}$$

7. (10 points) Find the equation of the tangent line to the function $f(x) = \sqrt{x}$ at $x = 100$ and use it to estimate $\sqrt{97}$.

$$f(100) = 10$$

$$f'(x) = \frac{1}{2} x^{-1/2}$$

$$\Rightarrow \quad f'(100) = \frac{1}{2} \cdot 100^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{100}} = \frac{1}{20}$$

Tangent line :

$$y - 10 = \frac{1}{20}(x - 100)$$

$$\Rightarrow \quad y = 10 + \frac{1}{20}(x - 100) = L(x)$$

So $\sqrt{97} = f(97) \approx L(97) = 10 + \frac{1}{20}(-3)$

$$= 9.85$$