West Chester University Mathematics Colloquium

Presents

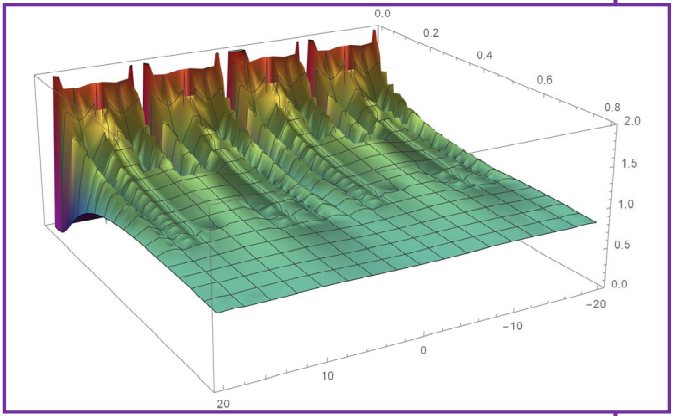
James McLaughlin

West Chester University

Vanishing Coefficients in the Series Expansion of Lacunary Eta Quotients

Time & Location
Wednesday, November 8, 2023
3:30 pm - 4:30 pm
25 University Avenue, Room 162

Dr. James McLaughlin is a Professor of Mathematics at West Chester University. He completed his Ph.D. at the University of Illinois in 2002 and has worked at WCU since 2005. Dr. McLaughlin is the author of over 50 papers and the book "Topics and methods in q-series."



Abstract:

For $|q| < 1$, define

$$(q;q)_\infty := (1-q)(1-q^2)(1-q^3)\cdots$$
$$f_1 := (q;q)_\infty \qquad f_j := (q^j;q^j)_\infty$$

Notice that the series expansion for f_1,

$$f_1 = (q;q)_\infty = 1 - q - q^2 + q^5 + q^7 - q^{12} - q^{15} + q^{22} + q^{26} - q^{35} - q^{40} + q^{51} + q^{57} - q^{70} - q^{77} + q^{92} + q^{100} - q^{117} - q^{126} + q^{145} + q^{155} - q^{176} - q^{187}\ldots$$

has most coefficients equal to 0, and in fact satisfies the definition of being *lacunary*.

Definition. The series $\sum_{n=0}^{\infty} c(n)q^n$ is *lacunary* if

$$\lim_{x\to\infty} \frac{|\{n \mid 0 \le n \le x, c(n) = 0\}|}{x} = 1.$$

Serre showed that f_1^r is lacunary for an even positive integer r if and only if $r \in \{2, 4, 6, 8, 10, 14, 26\}$.

Han and Ono proved the following theorem:

Theorem 2.1. *(Han and Ono 2011) Define the sequences $\{a_n\}$ and $\{b_n\}$ by*

(2.1)

$$f_1^8 =: \sum_{n=0}^{\infty} a_n q^n, \quad \frac{f_3^3}{f_1} =: \sum_{n=0}^{\infty} b_n q^n, \quad \text{where } f_i := \prod_{n=1}^{\infty}(1-q^{in}), \quad i \in \mathbb{Z}^+.$$

Then

(2.2)
$$a_n = 0 \iff b_n = 0.$$

Moreover, we have that $a_n = b_n = 0$ precisely for those non-negative n for which $ord_p(3n+1)$ is odd for some prime $p \equiv 2 \pmod 3$.

In this situation we say that f_1^8 and f_3^3/f_1 have *identically vanishing coefficients*.

The present talk presents the results of an in-depth investigation by the speaker and his collaborators (Tim Huber (University of Texas, Rio Grande Valley) and Dongxi Ye (Sun Yat-sen University, Guangdong, People's Republic of China) into the topic of lacunary eta quotients with identically vanishing coefficients (an *eta quotient* being a finite product of the form $\prod_j f_j^{n_j}$, for some $j \in \mathbb{N}$ and some $n_j \in \mathbb{Z}$, with a product with all $n_j > 0$ being termed an *eta product*).

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